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String Dualities from Matrix Theory

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We suggest that the (2,0) six dimensional field theory compactified on $S^1 \times K3$ is the Matrix model description of both M-theory on $K3$ and the Heterotic string on T^3. This proposal is different from existing proposals for the Heterotic theory. Different limits of the base space geometry give the different space-time interpretations, making M-theory/Heterotic duality manifest. We also present partial results on Heterotic/F-theory duality.

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1. Introduction

In the last few months a significant amount of evidence has accumulated in support of the conjecture of Banks, Fischler, Shenker and Susskind on the non-perturbative formulation of M-theory[1]. The conjecture, however, needs to be extended when describing M-theory compactified on a 4-torus down to seven dimensions. The reason is that the "Super-Yang-Mills (SYM) on the dual manifold" prescription guarantees that the extreme IR of the base-space will be roughly correct, but does not provide any information about its UV. The details of the extension were discussed in [2,3], where it was suggested that M-theory on a 4-torus is given by the large N limit of the (2,0) supersymmetric field theory compactified on a 5-torus.

As it is misleading to begin with the SYM prescription, our starting point will be the (2,0) field theory. The (2,0) theory is discussed in [32-34,5]. In this paper we discuss compactifications to seven and eight dimensions that have 8 linearly realized supersymmetries (in the infinite momentum frame). These are M-theory on $K3$, the Heterotic string on T^3 and T^2 and F-theory on $K3$. We obtain such a theory by compactifying the (2,0) theory on a 5-dimensional base-space that breaks half the supersymmetry. The manifold which we use is the natural candidate $S^1 \times K3$. We will refer to this manifold as the base-space.

In different degenerations of the $S^1 \times K3$ base-space, we obtain different spacetime interpretations. When the manifold degenerates to a 4 dimensional base-space in different ways we obtain M-theory on $K3$ and the Heterotic theory on T^3. As these configurations are smoothly connected, and we can follow the transition from one limit to another, the duality between these theories is manifest.

M-theory on $K3$ is described, in the base-space IR, by a SYM on another $K3$, with additional degrees of freedom. For the Heterotic theory on T^3 our proposal differs from existing proposals[10,11]. In this case the base-space $K3$ degenerates in a complicated way, which is well-defined only in some limits. We also discuss the origin of the fermions and 8-branes of [10-14] in our picture. This is discussed in section 2.

When the manifold degenerates, in a particular way, to a 3 dimensional manifold we obtain the Heterotic string on T^2 and F-theory on $K3$. For this duality[8] we obtain only partial results, as we can obtain F-theory only in a limit of its moduli space. In the general case we encounter a strongly coupled SYM theory, which limits our analysis. Again our proposal for the Heterotic string is different then existing proposals. For F-theory, in a certain limit, our proposal is an extension of the construction in [6] of the type IIB string. This is presented in section 3.

Many of the results in this paper were derived independently by P. Horava, and were reported in [4]. As we completed this paper, we received a paper that has some overlap with the results in section 2 [36].

2. M-Theory on $K3$ and Heterotic strings on T^3

Given the (2,0) theory on any manifold, spacetime is obtained only in certain limits of the base space geometry, when the (2,0) theory can be approximated by a Kaluza-Klein reduction to a semi-classical SYM theory [3]. In this section we consider the spacetime interpretations of the (2,0) theory on $S^1 \times K3$ in the limits of the base space geometry in which one of the dimensions is much smaller than the others. The Kaluza-Klein reduction is then a 4+1 SYM on a resulting 4-manifold (if such an object exists and is not too singular), and the space-time that emerges has seven non-compact dimensions. The resulting theory has 8 linearly realized supersymmetries which implies that the spacetime theory has a total of 16 supersymmetries. A natural conjecture is that this model defines the Heterotic string on T^3 or M-theory on $K3$. We show below that we obtain these two spacetime theories as different limits of the geometry of the base space $S^1 \times K3$.

2.1. M-Theory on $K3$

We begin by obtaining M-theory on $K3$ in Matrix theory [16]. Let us denote the size of the S^1 by Σ_1 and the volume of the $K3$ by V. The limit of M-theory on a large $K3$ is obtained by

$$\Sigma_1, V \to 0$$

$$\tfrac{\Sigma_1}{V} \ fixed. \tag{2.1}$$

The second requirement guarantees that the eleven dimensional Planck scale is fixed.

When Σ_1 is much smaller than any length scale of the $K3$, the IR of the base-space theory is approximated by a Kaluza-Klein reduction on the S^1, which is a weakly coupled SYM on $K3$[1] [23] (the gauge coupling of the this SYM is $g^2 = \Sigma_1$). Via T-duality [26,25] we then obtain a description of the moduli space of the theory in terms of 0-brane moving on the dual $K3$, which is the physical space-time $K3$.

In the case of orbifold limits of $K3$, the requirement that S^1 is smaller than any length scale of $K3$ cannot be satisfied, as there are 2-cycles of zero size. This results in the appearance of additional degrees of freedom in the effective 4+1 SYM. These are related to the 32 fermions that appear in [10-14] and will be discussed further in section 2.1.1. For now we restrict our attention to degrees of freedom that live in the bulk, far away from any orbifold points.

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[1] It was suggested in [11] that a related 4+1 SYM might have non-zero instanton number. We do not discuss this issue here [22].

For convenience, let us work in the limit that $K3$ is the orbifold T^4/Z_2, whose sizes are $\Sigma_2, ..\Sigma_5$. The relation of the spacetime parameters to the SYM parameters are similar to those of M-theory on a 4-torus, and are given in [2,15]. These are

$$L_i{}^2 = \frac{2\pi RV}{\Sigma_1 \Sigma_i{}^2}$$
$$L_p{}^6 = \frac{R^3 V}{(2\pi)^3 \Sigma_1} \quad . \tag{2.2}$$

Where L_i (i= 2,...,5) are the spacetime lengths and L_p is the eleven dimensional Planck length.

In the limit when $g^2 \to 0$ ($\hbar \to 0$) the theory becomes semi-classical and the Wilson lines define a moduli space. This moduli space is interpreted as the classical compactification manifold in spacetime. The weakly coupled 4+1 SYM description of this space is equivalent to the description of 0-branes moving on this manifold, as can be shown by a T-duality. This description is valid when this spacetime manifold is much larger than L_p.

Another check that we have identified correctly the Matrix theory is to reproduce the moduli space of M theory on $K3$. As is often the case in the infinite momentum frame, modifications to the ground state of the theory are obtained by modifying the Hamiltonian. In our case we can modify the base space geometry. The different choices of base space geometry should give the spacetime moduli space.

We are therefore interested in 5 dimensional manifolds that break half the supersymmetry. First there are several discrete choices [2]. These choices are either $K3 \times S^1$, or $K3 \times S^1$ modded out by some discrete group. It is interesting to see whether the latter choices give new theories in seven dimensions [3], but here we focus on the first option[4]. Once we have made all the discrete choices, the only parameters are those which define a metric on $S^1 \times K3$ with an $SU(2)$ holonomy. The only parameters of such a metric are the size of the S^1 and a choice of an Einstein (Ricci flat) metric on $K3$ (there can be no components of the metric that mix the $K3$ and the S^1). The Moduli space is therefore locally $SO(3,19)/(SO(3) \times SO(19)) \times R^+$. This is the correct moduli space of M-theory on $K3$ (and of the Heterotic string on T^3) [7].

The model also has enhanced gauge symmetries at the correct points of moduli space. If the base-space has certain singularities, then the T-dual $K3$ has similar singularities[5]. It

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[2] We are indebted to P. Aspinwall for supplying us with this information.

[3] The counting of Moduli seems to be different. See also [31]

[4] To completely define the theory we need also to choose a spin structure [30]. We are not sure how that affects our discussion.

[5] We are indebted to S. Kachru for a discussion of this point.

was shown in [17,16] that the $N \to \infty$ Matrix theory then has the additional states which make up the additional gauge bosons.

We conclude this section in a computation which will be useful later. To get the correct result after T-dualizing to the zero-brane picture we need to specify the correct boundary conditions on the gauge fields. Here we state the boundary conditions which are derived from this requirement and later we will use them to obtain the boundary conditions in the Heterotic limit. The boundary conditions on the gauge fields are [17,19,21,22]

$$A_\mu(x) = -\gamma A_\mu(\widetilde{x})\gamma, \ \mu = 1..4$$

$$A_0(x) = \gamma A_0(\widetilde{x})\gamma \tag{2.3}$$

$$Y_i(x) = \gamma Y_i(\widetilde{x})\gamma$$

where γ is some matrix such that $\gamma^2 = 1$, and $\widetilde{x}$ is the image of x under the Z_2 action. The Matrix model includes all choices of γ [17]. Note that under these boundary conditions, the instanton number[6] remains invariant. This is consistent with the product structure of the base space - the orbifold group Z_2 does not act on the small circle.

We are interested in lifting the boundary conditions to the (2,0) theory. Since we do not know how to write a theory of B fields in a $U(N)$ invariant way, lifting the boundary conditions is justified only for $N = 1$. In this case we can take $\gamma = 1$. Near an ALE point in spacetime such a projection on a zero-brane corresponds to a 2 brane wrapped on the shrunken cycle in the ALE [17,18]. Since $A_\nu = B_{1\nu}$ the boundary conditions we obtain are

$$B_{\mu\nu}(x) = \chi(\mu)\chi(\nu)B_{\mu\nu}(\widetilde{x})$$

$$Y^i(x) = Y^i(\widetilde{x}) \tag{2.4}$$

where $\chi(\mu) = -1$ for $\mu = 2, 3, 4, 5$ and $\chi(\mu) = 1$ for $\mu = 0, 1$. We will return to these boundary conditions later.

2.1.1 Additional Degrees of Freedom

As we have seen in the Matrix description of M-theory on T^4 [3], the process of obtaining the world-volume description from a spacetime picture is incomplete. In general the spacetime description probes only the IR of the base space theory, and in order to define it one has to add UV information. In the present case, the spacetime-based prescription of

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[6] The instanton number in the 4+1 SYM is identified with momentum in the small circle direction.

"T-dualizing 0-branes on a $K3$" fails to capture the complete set of IR degrees of freedom if not done carefully [21,22].

To demonstrate this, let us discuss the (2,0) theory along its flat directions, where we know how to write it down. In the T^4/Z_2 limit of the $K3$ we have 16 shrunken 2-cycles and another 10 finite 2-cycles that are associated with the bulk of $S^1 \times T^4/Z_2$. The 10 cycles in the bulk become the gauge field after we reduce on Σ_1. The other 16 cycles, however, also contribute massless degrees of freedom. These are equal to $\int B$ on the shrunken 2-cycles. In fact, these are the 16 chiral bosons of the Heterotic string in [27], and will be identified with the 32 fermions in the Matrix picture of the Heterotic string on T^3 which we discuss shortly. Whether they will remain massless when we take $\Sigma_1 \to 0$ depends on their periodicity along the S^1 direction. In the Heterotic case [10] half the fermions are periodic along the S^1 and half are anti-periodic. Borrowing this result for our case (we did not derive it from the (2,0) theory) suggests that half of them remain massless in this limit.

On the other hand, the analysis of 0-branes moving on the physical T^4/Z_2 does not require these additional degress of freedom, and T-dualizing this picture gives only the Wilson lines in the bulk.

There are additional degrees of freedom that the 0-branes might miss. Suppose we slightly blow up the orbifold. This is given [19,20] by a small deformation of the 0-brane Hamiltonian. The masses of strings that stretch between 0-branes are likewise slightly perturbed. In our suggestion the situation is different. Here we have also slightly blown up certain singularities and there are now configurations of the B-field in which it varies on the small cycle. These are new massive degrees of freedom whose mass scale is inversely proportional to the size of the small cycles. It would seem that the local picture of 0-branes near a slightly blown-up ALE point misses these states, which exist in our proposal.

2.2. Heterotic Theory on T^3

2.2.1 Heterotic vacuum with $SU(2)^{16}$ Enhanced Gauge Symmetry

The case in which $K3 = T^4/Z^2$ (which has 16 A_1 singularities) is the easiest to analyze. This configuration is the one that is most closely related to the configuration of [10,11]. Let us pick one dimension of T^4, say Σ_5, and take it to zero, as well as the volume of the remaining space $V = \Sigma_1\Sigma_2\Sigma_3\Sigma_4$. Again, we take V and Σ_5 to zero at a fixed ratio.

After the Kaluza-Klein reduction on Σ_5 we obtain a SYM on $S^1 \times (T^3/Z_2)$ with a coupling $g^2 = \Sigma_5$ and some boundary conditions on the gauge fields. These are obtained from the boundary conditions on the B fields. For N=1 we can check this explicitly. The

Kaluza-Klein reduction is obtained by defining $A_\mu = B_{5\mu}$, $\mu = 1, 2, 3, 4$ and using (2.4) (for N=1). The boundary conditions one obtains are

$$A_{0,1}(\sigma, \sigma^i) = -A_{0,1}(\sigma, -\sigma^i)$$

$$A_a(\sigma, \sigma^i) = A_a(\sigma, -\sigma^i), \ a = 2, 3, 4 \qquad (2.5)$$

$$Y^i(\sigma, \sigma^i) = Y^i(\sigma, -\sigma^i)$$

which are the same as in [10]. Note also that the instanton number reverses its sign under the Z_2 action, which means that these boundary conditions give the unique correct extension to the 5-dimensional manifold $S^1 \times T^4/Z_2$. [7]

We have obtained an Heterotic string theory on T^3, and we can write its parameters in terms of $g^2, \Sigma_{1,2,3,4}$. It is more instructive, however, to write it in terms of the M-theory on $K3$ parameters (2.2). Doing so, one obtains

$$T_{string} = \frac{L_2 L_3 L_4 L_5}{(2\pi)^5 L_p^6}$$
$$\lambda_7^4 = \frac{(L_2 L_3 L_4 L_5)^3}{(2\pi)^2 L_p^{12}} \qquad (2.6)$$

which are the seven dimensional Heterotic/M-theory duality relations [7]. One can also reproduce more detailed formulas that relate the radii of the $K3$ to those of the T^3[24].

2.2.2. A Conjecture Regarding the $E_8 \times E_8$ Case

We are interested in the Heterotic string on T^3 in its M-theory limit. *i.e.* when it is M-theory on $S^1/Z_2 \times T^3$. We therefore expect to see a well defined moduli space of this form only when the space-time gauge symmetry is $E_8 \times E_8$, or a subgroup of it. We are interested not only in the moduli space, but also in the masses of some of the modes when we go along the flat directions. These are important in order to reproduce graviton scattering.

The picture that we suggest is very similar to that of Vafa and Morrison [29]. Let us check the case in which the base-space $K3$ has two E_8 singularities. The $K3$ can then be written as an elliptic fibration over P^1. On the P^1 there are two singular fibers which contain the E_8 singularities and four additional singularities (where the fiber but not the K3 degenerate). We are interested in the limit in which a pair of the additional singularities approach each E_8 locus. In that case the base becomes a long thin cylinder capped in the

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[7] As was explained in [17], the state with N=1 corresponds to a solitonic state in the M-theory. We are now in position to see how the gauge boson changes from a perturbative state in the Heterotic string to a solitonic state in M-theory.

vicinity of the E_8 singularity. Throughout the cylinder, as long as we are away from the singularities, the fibre has a constant complex structure parameter.

We are interested in reducing the (2,0) theory on the small circle which is a part of the cylindrical base of the elliptic fibration. We take therefore all the other dimensions of the base space to be of the same order of magnitude, and larger than this small circle. Note that the size of the fibre is a parameter of the theory (unlike in F-theory). In this configuration the (2,0) theory has a mass gap and we can perform a Kaluza-Klein reduction on the small circle. The base space of the resulting 4+1 SYM looks like $S^1 \times T^2 \times I$ where the first S^1 is outside the K3, the T^2 is the fibre and I is an interval, which is what is left from the cylinder after the Kaluza-Klein reduction. On this space we have a weakly coupled gauge theory with some boundary conditions on the gauge fields and matter fields. We can now have four Wilson lines on this space which give us the four compact space coordinates.

We do not know how to calculate the boundary conditions in this picture, so we can not verify it. Let us note, however, that the base space of this SYM, $S^1 \times T^2 \times I$, is metrically flat. This means that the higher momentum modes and the energy of W's, which we integrate out when we calculate graviton scattering, are spaced at constant intervals in a way that is similar to the spacing of the stretched strings in the 0-brane picture. It is possible therefore that it reproduces correctly the graviton scattering amplitude.

It is interesting to compare this picture to the current suggestion of the Heterotic string on T^3 [10,11]. In that suggestion the base space is always $S^1 \times (T^3/Z_2)$ (perhaps with a non-trivial metric [11]) and the fermions are not associated with the singularities but rather can move about. The fundamental domain of the Z_2 action can be taken to be $S^1 \times T^2 \times I$ which is the same base space that we obtained. From the point of view of the base space the only difference is the resolution of the singularities at the boundaries of the fundamental domain.

Away from the $E_8 \times E_8$ loci, we obtain the full $K3$. For a general shape of the $K3$ our suggestion differs from [10,11]. In general there is no well defined way to perform a Kaluza-Klein reduction and obtain a SYM on some 4-manifold. Even if we are able, at certain limits, to pick a small circle in the $K3$ and reduce on it, the resulting 4-manifold would be highly singular. It is not clear in that case how a SYM can be defined on such a manifold.

Another important difference is the way that the fermions are treated. In our picture the fermions are to be understood as fermionization of the bosons $\int B$ over shrunken cycles. As such they are localized at the singularities and are not allowed to move. Enhanced symmetry is obtained in a geometric way in which the mixing of the compact space parameters and the $E_8 \times E_8$ Wilson lines is apparent.

More important is the fact that we do not need to add these fermions by hand [12]. They are automatically provided by the (2,0) definition of the theory. At no point of the discussion do we need to take a circle, in the Matrix description of M-theory on T^4, orbifold it and add 8-branes. Rather the 8-branes are generated in the effective space-time by the existence of additional degrees of freedom in a specific degeneration of the (2,0) base-space. We know that there are 8-branes only through the existence of the 32 fermions. When we calculate any low energy scattering the fermions contribute to the scattering amplitude such that a low energy observer interprets the result as the existence of 8-branes.

When we blow up a singularity, to break the space-time gauge symmetry, we still have the 32 fermions, but now we have many more degrees of freedom. There are finite energy configurations of the B fields in which they vary on the cycle which we have blown up. This is again a departure from the current suggestion for the Matrix description of the Heterotic string, and it demonstrates again the problems that arise when we take the space-time picture and try to extrapolate it to the base-space.

2.3. Duality

To summarize, both M/$K3$ and Het./T^3 are described by the same model. In one limit of the geometry of the base-space we obtain the a description of the weakly coupled low-energy of the Heterotic string and in another limit that of M theory on $K3$. The transition between these two limits, as expected, goes through a region in which the compact part of space-time is not well defined.

3. F-theory on $K3$ and Heterotic on T^2

Since we have found Heterotic(T^3)-M($K3$) duality as a simple geometric relation in the (2,0) field theory, it is interesting to ask whether we can do the same for the Heterotic/F-theory duality in eight dimensions [8]. The Matrix model of the Heterotic theory is similar to the one we presented in the previous section, and we present here only a partial analysis of the IIB side. More precisely, in the region of parameters where we can analyze the base-space theory reliably, we obtain F-theory compactification on $K3 \times S^1$ down to seven dimension. Since the radius of the circle will be much smaller then that of the F-theoretic base of the $K3$, we never reach the 8 dimensional F-theory. We comment below on possible ways of analyzing the 8 dimensional limit.

A point of notation. In places that the notation might be ambiguous we denote by a subscript F quantities from F-theory. For example $K3_F$ denotes the elliptically fibered $K3$ that defines the F-theory compactification. Similarly a subscript M denotes M-theory quantities, and a subscript B denotes quantities that relate to the base-space on which we compactify the (2,0) theory.

3.1. The Heterotic Theory

We begin with the Heterotic theory on T^2. We take $K3_B$ to be elliptically fibered. Let $\Sigma_{2,3}$ denote the sizes of the fibre and $\Sigma_{4,5}$ the sizes of the base of the elliptic fibration. We suggest that the Heterotic spacetime will be obtained in the limit in which the base goes to zero.

For the purposes of scaling we concentrate on the case of $K3 = T^4/Z_2$. We work in the following limit of the base space geometry

$$\Sigma_{4,5} \to 0$$

$$\frac{\Sigma_5}{\Sigma_4} \ll 1, \ fixed. \tag{3.1}$$

$$\Sigma_{1,2,3} \ fixed$$

Because of this hierarchy we first reduce the (2,0) theory on Σ_5. The result is a SYM theory in 4+1 dimensions with a coupling $g^2 = \Sigma_5$. We then reduce on the circle of length Σ_4. In this limit we obtain 3+1 SYM with a coupling $g^2 = \frac{\Sigma_5}{\Sigma_4}$ which is weak. The moduli space of this SYM theory describes the resulting spacetime. The spacetime dimensions are given in terms of the SYM quantities as follows:

$$l_k{}^2 = \frac{2\pi R \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{\Sigma_5 \Sigma_k{}^2}, \quad k = 1, 2, 3$$

$$l_4{}^2 = \frac{2\pi R \Sigma_1 \Sigma_2 \Sigma_3}{\Sigma_4 \Sigma_5} \tag{3.2}$$

$$l_{p,11}{}^6 = \frac{R^3 \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{(2\pi)^3 \Sigma_5}$$

Where $l_{p,11}$ is the eleven dimensional Planck length, l_4 is a dimension that decompactifies in the limit of a vanishing base.

The resulting spacetime is therefore 8 dimensional and describes the M-theory compactified on $S^1/Z_2 \times T^2$. The relevant quantities that measure the decompactification of the 8th dimension are $l_4/l_{p,11}$ or $l_4/l_{p,8}$ where $l_{p,8}$ is the eight dimensional Planck scale. Using (3.2) we obtain

$$\frac{l_4^2}{l_{p,8}^2} = \frac{(\Sigma_1 \Sigma_2 \Sigma_3)^{\frac{2}{3}}}{\Sigma_4 \Sigma_5}. \tag{3.3}$$

Therefore, in the limit (3.1) we indeed approach an eight dimensional spacetime.

As expected, Σ_1, the base space circle, is inversely proportional to the spacetime orbifold length, l_1. A large base space circle corresponds therefore to a weak Heterotic coupling. The spacetime 2-torus is dual to the base space torus, which is the elliptic fiber of the $K3_B$ surface.

As before, the manifold which we obtain after we shrink part of $K3_B$ can be very singular. It is not clear how to define a SYM theory on it, other than as the Kaluza-Klein reduction above. This was an issue that we have already seen in the case of the Heterotic theory on T^3, and the discussion there applies here as well.

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3.2. F-theory

The Heterotic string in eight dimension has a dual description in terms of F-theory compactified on $K3_F$. The base of $K3_F$ is a compactification manifold for type IIB theory, and the fibration describes $SL(2, Z)$ monodromies on the base. The Heterotic coupling appears in this context roughly as the area of the base (the more precise statement is given below).

Let us first discuss what is our goal. We will be able to attain it only partially. We are interested in a Matrix description of a certain vacuum of type IIB string. In [6] the IIB string theory was constructed in terms of an interacting fixed point field theory in 2+1 dimensions [5]. If we want to obtain the IIB string on a circle, we expect to have (probably only in the IR) a family of 2+1 fixed point theories that have a prefered scalar. As we change the vev of the scalar we sweep out the additional circle.

The way to obtain such a picture is to start with M-theory on T^3 and take two of the circles of the dual torus to infinity. We can then obtain this picture by doing a Kaluza-Klein reduction on the small circle. The preferred scalar is then the Wilson line along this small circle. Another way of rephrasing this construction is the following. SYM on the 3-torus is analogous to a IIB 3-brane wrapping a 3-torus. When one of the circles of the two-torus goes to zero the IR is better described by T-dualizing this smaller circle to a IIA 2-brane wrapping the two large circles.

The limit which we now discuss is the following

$$\Sigma_1, \Sigma_4, \Sigma_5 \to 0$$

$$\Sigma_1 \ll \Sigma_4, \Sigma_5$$

$$\frac{\Sigma_1}{\Sigma_4 \Sigma_5} \to \infty \qquad (3.4)$$

$$\Sigma_2 \ll \Sigma_3 \ both \ held \ fixed$$

(the reason for taking this limit is to obtain a regime in which F-theory is described reasonably well by 10D supergravity, as we will see later).

Given this hierarchy, we first reduce the (2,0) theory on Σ_1, obtaining a SYM theory in 4+1 dimensions, with a coupling $g^2 = \Sigma_1$, compactified on a $K3_B$ surface. As discussed above the spacetime interpretation of this theory is M-theory on a $K3_M$. In the limit (3.4) we get a particular degeneration of $K3_M$, in which the fibre is much smaller than the base. This is the immediate generalization of the description above for the M-theory on a 3-torus giving IIB on a circle. In our case, this is the appropriate degeneration that yields the F-theory description [8].

To analyze the moduli space, it is again convenient to use an analogue model, which is the following. SYM on a $K3_B$ manifold has an analogue model which is a IIA 4-brane wrapping the $K3$. We are interested in a description in which we have a 2-brane wrapping the fibre. We can accomplish this by performing a T-duality transformation that takes the volume of the $K3$ to its inverse, and the wrapped 4-brane to a 0-brane [25]. We then do T-duality on each fibre, or go to the mirror $K3$ [35], to obtain a description in terms of 2-branes wrapping the fibre. Out of these three configurations (wrapped 4-brane, 0-branes or wrapped 2-brane) we should take the one that describes the IR behavior more accurately. Since we started with a large fibre and a small base, the wrapped 2-brane has a large base and a large fibre and is the correct description. We obtained a picture of an elliptically fibered $K3_F$, with a 2+1 SYM wrapping the fibres. This is exactly what we would expect as world volume IR description of the IIB string. This procedure is essentially going to the IIB limit by shrinking the fibre in M-theory on $K3$.

Two points are important. One is that this picture only captures the IR, which is sufficient for our purposes. The second is that we did our computation in weak coupling (of the SYM of $K3_B$), which means that it is reliable. This was done by taking Σ_1 to be much smaller than any other scale. The down side, as we will see shortly, is that the base-space of F-theory is much larger than the additional dimension which grows (when we shrink the two dimensions in M-theory) so we are not in the eight dimensional F-theory limit. However, both the new dimension which grows and the base of $K3_F$ are much larger than the 10 dimensional IIB Planck length, so it is a regime in which we require to have a well defined moduli space.

One can be much more precise about the parameters of the resulting F-theory (we will work in the orbifold limit). The M-theory on $K3$ parameters were obtained above, in (2.2). In the limit we are considering in the present context, the $K3_M$ fibres go to zero. Whenever a spacetime torus of lengths L_2, L_3, shrinks in M-theory, we get type IIB theory with a large circle of length [9]

$$L_F = \frac{(2\pi)^3 L_p{}^3}{L_2 L_3} = \sqrt{\frac{2\pi R \Sigma_1 \Sigma_2 \Sigma_3}{\Sigma_4 \Sigma_5}} \to \infty \tag{3.5}$$

And the size of the base space of $K3_F$ is

$$L_4 L_5 = \frac{2\pi R \Sigma_2 \Sigma_3}{\Sigma_1} \to \infty. \tag{3.6}$$

so both the base-space and the additional circle go to infinity. In particular they grow relative to IIB 10 dimensional Planck scale $l_{p,10}^4 \propto R^2 \Sigma_2 \Sigma_3$ so we expect to have a good moduli space.

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The coupling constant of the type IIB theory is the complex structure of the base space fiber, $\frac{\Sigma_2}{\Sigma_3}$. More generally the shape of the torus on which the 2+1 SYM lives changes as we go around the base space. This is expected in view of [6].

Another quantity which we can calculate is the size of the base relative to the size of the additional circle

$$\frac{L_F{}^2}{L_4 L_5} = \frac{\Sigma_1^2}{\Sigma_4 \Sigma_5} \ll 1 \tag{3.7}$$

The additional circle is much smaller than the base, and the theory does not approach an eight dimensional theory. However, again, since both these sizes are larger than $l_{p,10}$ we expect to see them in the moduli space of the base-space theory.

The eight dimensional limit can be reached if we take the circle Σ_1 to be larger than the $K3_B$ base, unlike the limit in (3.4). We do not know how to get a moduli space which corresponds to the F-theory base space in that case. Reduction on the Σ_1 first gives natural variables by which to describe the base of $K3_F$, but this is justifiable only when Σ_1 is the smallest length scale. In the eight dimensional limit we need to reduce on $\Sigma_{1,4,5}$. For a generic $K3_B$ this yields a very singular object, but in simple cases (orbifolds for example) we obtain a 2+1 dimensional strongly coupled SYM theory. It may be that there is some D=3 duality by which one will be able to obtain the IIB space-time.

To summarize we see the base of elliptic fibration of the IIB string in the F-theory on $K3 \times S^1$ only in a specific regime. In that case it can be seen using the full 5+1 dimensional definition of the theory.

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